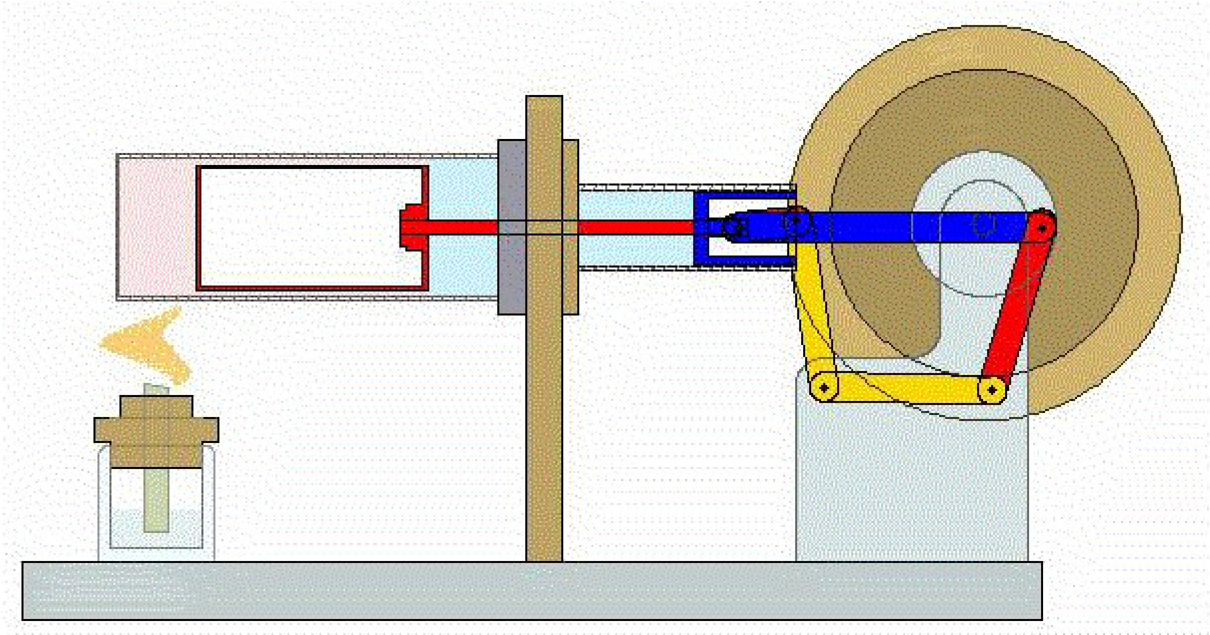
Heat engine, Carnot's engine & Refrigerator



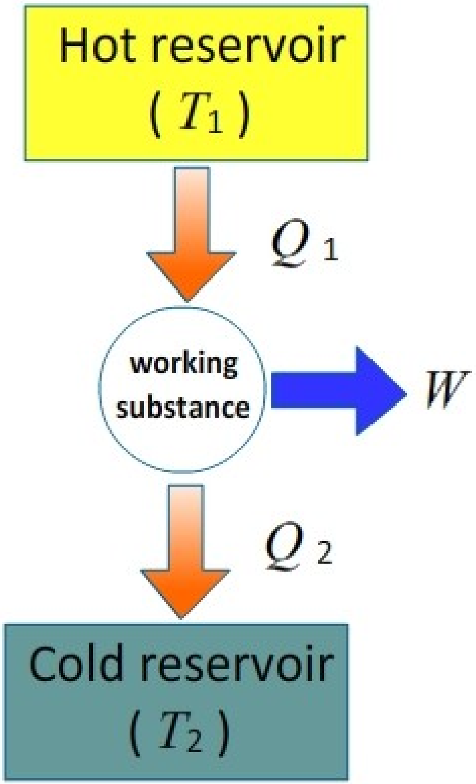
V Vijaykiran
M.Sc., B.Ed., NET
SIGMA Physics Resource Centre
http://www.sigmaprc.in

Heat engine

A device in which a system is made to undergo a cyclic process that results in conversion of heat to work

Efficiency (η) of a heat engine is defined as the ratio of work done by the system to the heat energy absorbed by it.

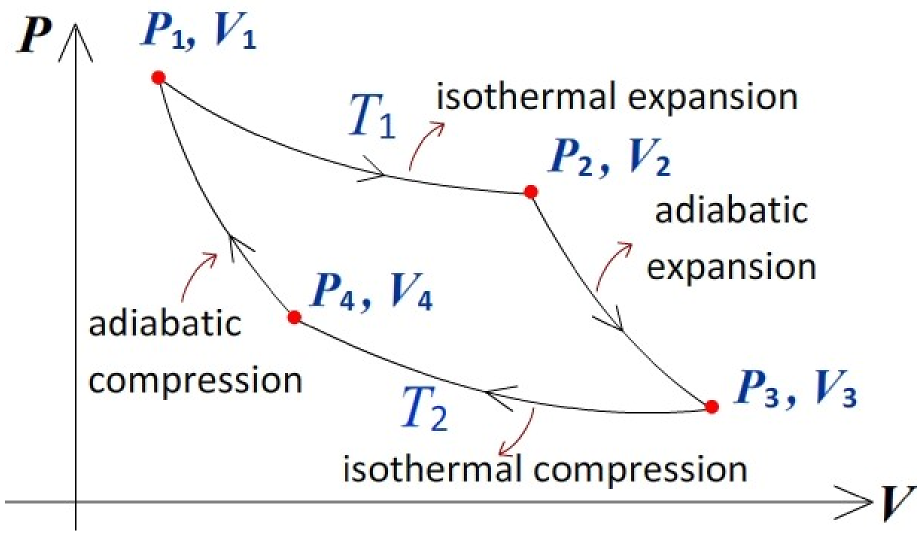
$$\eta = \frac{W}{Q}$$



Schematic diagram of a heat engine

Carnot's engine

An ideal heat engine in which the working substance is an ideal gas and the cycle consists of ideal isothermal and adiabatic processes. In each cycle of operation, heat is absorbed from a source or hot reservoir at high temperature (T_1), the system undergoes expansion and the unused heat is rejected to a sink or a cold reservoir at low temperature (T_2).



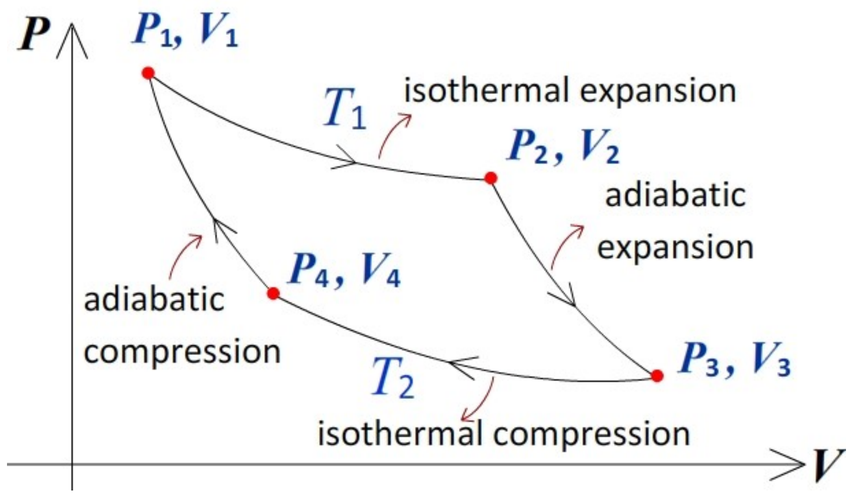
Each cycle consists of the following 4 steps

1. Isothermal expansion (T_1)
2. Adiabatic expansion (T_1 to T_2)
3. Isothermal compression (T_2)
4. Adiabatic compression (T_2 to T_1)

Efficiency of a heat engine is given by

$$\boxed{\eta = \frac{W}{Q} = 1 - \frac{T_2}{T_1}}$$

Thermodynamics



$$W_1 = nRT_1 \ln\left[\frac{V_2}{V_1}\right] \quad \text{— i}$$

$$W_2 = \frac{nR}{1-\gamma}(T_2 - T_1) \quad \text{— ii}$$

$$W_3 = nRT_2 \ln\left[\frac{V_4}{V_3}\right] \quad \text{— iii}$$

$$W_4 = \frac{nR}{1-\gamma}(T_1 - T_2) \quad \text{— iv}$$

$$W_{net} = W_1 + W_2 + W_3 + W_4$$

Substituting eqs (i) to (iv) in above relation

$$W_{net} = nR\left(T_1 \ln\left[\frac{V_2}{V_1}\right] + T_2 \ln\left[\frac{V_4}{V_3}\right]\right) \quad \text{— v}$$

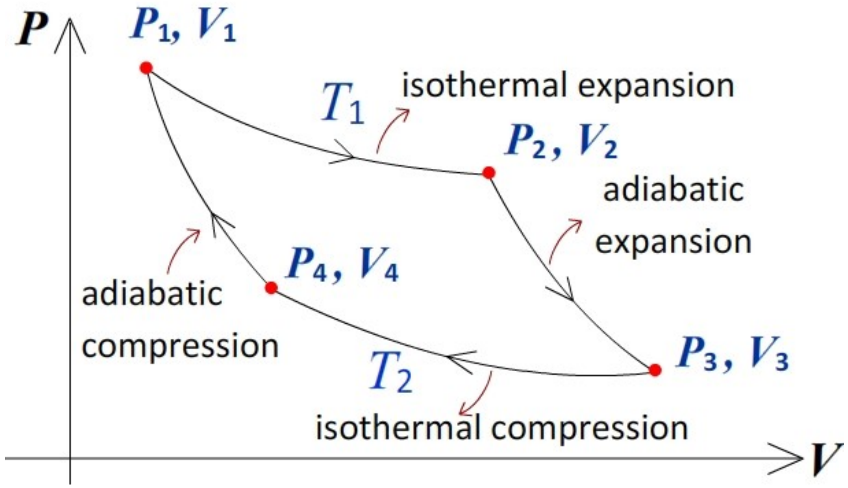
Heat absorbed by the system is given by

$$Q = nRT_1 \ln\left[\frac{V_2}{V_1}\right] \quad \text{— vi}$$

Using eqs (v) & (vi) in efficiency relation

$$\eta = \frac{nR\left(T_1 \ln\left[\frac{V_2}{V_1}\right] + T_2 \ln\left[\frac{V_4}{V_3}\right]\right)}{nRT_1 \ln\left[\frac{V_2}{V_1}\right]} \quad \text{— vii}$$

Thermodynamics



In adiabatic process $TV^{\gamma-1} = \text{const}$

For adiabatic expansion

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \Rightarrow \frac{T_1}{T_2} = \frac{V_3^{\gamma-1}}{V_2^{\gamma-1}} \qquad \text{viii}$$

For adiabatic compression

$$T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1} \Rightarrow \frac{T_1}{T_2} = \frac{V_4^{\gamma-1}}{V_1^{\gamma-1}} \qquad \text{ix}$$

From (vii) & (viii) we get $\dfrac{V_4}{V_3} = \dfrac{V_1}{V_2}$

Using this in eq(vi) we get

$$\eta = \frac{nR\left(T_1 \ln\left[\frac{V_2}{V_1}\right] + T_2 \ln\left[\frac{V_1}{V_2}\right]\right)}{nRT_1 \ln\left[\frac{V_2}{V_1}\right]}$$

$$\eta = \frac{\left(T_1 \ln\left[\frac{V_2}{V_1}\right] - T_2 \ln\left[\frac{V_2}{V_1}\right]\right)}{T_1 \ln\left[\frac{V_2}{V_1}\right]}$$

$$\boxed{\eta = \frac{(T_1 - T_2)}{T_1}}$$

- Efficiency of any heat engine is given by W/Q but that of a Carnot's engine is also given by $(T_1 - T_2)/T_1$
- Even in case of a Carnot's engine, efficiency of a heat engine is not equal to 1

Efficiency of heat engine cannot be 100% because this would imply that

$$\boxed{\eta = 1 - \frac{T_1}{T_2}}$$

- All the heat absorbed is converted to work which is not possible
- Temperature of surrounding is 0K which is not possible
- Temperature of source should be infinite which is not possible

Carnot's theorem :

(a) No heat engine working between a set of temperatures can be more efficient than a Carnot's engine working between the same set of temperatures.

(b) Efficiency of the Carnot engine is independent of the nature of the working substance i.e. $\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$

Heat pump (refrigerator)

It is a device that draws heat from a body (at lower temperature) and transfers it to surroundings (at a higher temperature) with external aid.

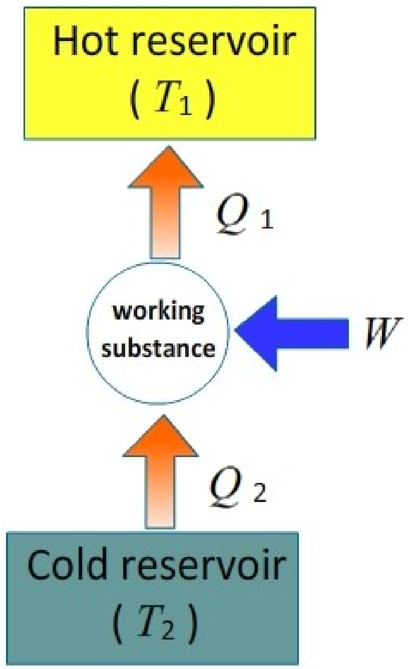
In a refrigerator the working substance, usually in gaseous form, goes through the following steps in each cycle of its operation

(a) sudden expansion of the gas from higher to lower pressure cools it and converts it into a vapour-liquid mixture

(b) vapour-liquid mixture absorbs heat from the region to be cooled and converts to vapour

(c) This vapour is heated due to external work done on it

(d) Heat is released by the vapour to the surroundings, bringing it to the initial state

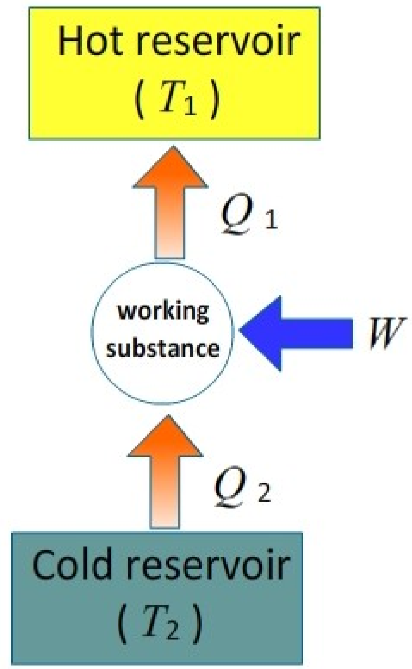


Heat pump (refrigerator)

Coefficient of performance of a heat pump

$$\alpha = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

α of a heat pump can never be infinite. This means that heat cannot be drawn out from a colder body on its own.



Note : α is NOT $1/\eta$

When a heat engine (of a given efficiency) is used as a refrigerator then calculate the values of T_C and T_H and then proceed to use them for refrigerator.

Second law of thermodynamics

<u>Kelvin-Planck statement</u>

No process is possible whose sole result is the absorption of heat from a reservoir and complete conversion of the heat into work.

This statement negates the possibility η being equal to 1

<u>Clausius statement</u>

No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.

This statement negates the possibility α being equal to infinity

The above two statements are completely equivalent.

While the first law of thermodynamics is based on the law of conservation of energy, the second law determines the direction of evolution of process.

Second law of thermodynamics – Heat engines and heat pumps

Second law of thermodynamics also implies that the efficiency of a heat engine cannot be 1 and the coefficient of performance of a refrigerator cannot be infinity.